

F

COLLECTING LIKE TERMS

Like terms are algebraic terms which contain the same variables to the same indices.

For example:

- $2xy$ and $-2xy$ are **like terms**
- a^2 and $-3a$ are **unlike terms** because the indices of a are not the same.

Algebraic expressions can often be simplified by adding or subtracting like terms. This is sometimes called **collecting like terms**.

$$\text{Consider } 2a + 4a = \underbrace{a + a}_{\text{"2 lots of } a"} + \underbrace{a + a + a + a}_{\text{"4 lots of } a}.$$

In total we have 6 lots of a , and so $2a + 4a = 6a$.

Example 12

Self Tutor

Simplify, where possible, by collecting like terms:

a $3x + 2x$

b $7a - 3a$

c $-2x + 3 - x$

d $3bc + bc$

e $2x - x^2$

a $3x + 2x$
 $= 5x$

b $7a - 3a$
 $= 4a$

c $-2x + 3 - x$
 $= -3x + 3$
{since $-2x$ and $-x$
are like terms}

d $3bc + bc$
 $= 4bc$

e $2x - x^2$ is in simplest form
{since $2x$ and $-x^2$ are unlike
terms}

EXERCISE 2F

1 Simplify, where possible, by collecting like terms:

a $3 + x + 5$

b $8 + 7 + x$

c $p + 3 + 7$

d $12 + a + a$

e $b + 3 + b$

f $b + b$

g $2x + x$

h $a + 3 + a + 7$

i $x + 3x$

j $3x - 2x$

k $3x - x$

l $a^2 + a^2$

m $7x + 3$

n $2x^2 + x^2$

o $17x - 7$

p $17x - x$

q $3b^2 - b^2$

r $2ab + 3ab$

s $g + g + g$

t $9b - 7b - 2$

2 Simplify, where possible:

a $11n - 11n$

b $11n - n$

c $11n - 11$

d $3ab + ba$

e $xy + 2xy$

f $2p^2 - p^2$

g $3a + 2 + a + 4$

h $2a + 3a + 4a$

i $b + 3 + 2b + 4$

j $3xy + 4yx$

k $2a + b + 3a + b$

l $3a^2 + a + a^2 + 2a$

m $3x + 2x - x$

n $n + 2n - 3n$

o $ab + b^2 + 2ab + b^2$

p $3x + 7x - 10$

q $3x + 7x - 10x$

r $3x + 7x - x$

s $r + r + 2r^2$

t $x^2 + x + 2$

u $3 + 6y - 1 + 2y$

3 Simplify, by collecting like terms:

a $3x + 8x$

b $3x - 8x$

c $-3x + 8x$

d $-3x - 8x$

e $5a + a$

f $5a - a$

g $-5a + a$

h $-5a - a$

i $m^2 + 2m$

j $-3d - 5d$

k $-3d + 5d$

l $3d - 5d$

m $b + 2b - 3$

n $t - 3t - 2t$

o $-6g - g$

p $4m - 7m + 1$

q $a + 2 - 3a$

r $-2b - -3b$

s $3b - b$

t $3b - -b$

u $x - -2x$

Example 13

Self Tutor

Simplify, by collecting like terms:

a $2 + 3a - 3 - 2a$

b $x^2 - 2x + 3x - 2x^2$

$$\begin{aligned} \mathbf{a} \quad & 2 + 3a - 3 - 2a \\ & = 3a - 2a + 2 - 3 \\ & = a - 1 \end{aligned}$$

{ $3a$ and $-2a$ are like terms,
 2 and -3 are like terms.}

$$\begin{aligned} \mathbf{b} \quad & x^2 - 2x + 3x - 2x^2 \\ & = x^2 - 2x^2 - 2x + 3x \\ & = -x^2 + x \end{aligned}$$

{ x^2 and $-2x^2$ are like terms,
 $-2x$ and $3x$ are like terms.}

4 Simplify, where possible:

a $a + 2 - 2a - 5$

c $ab + 2 - 3ab - 4$

e $3x - 4 + 4 - 4x$

g $-3n + 2 + n - 7$

b $a - 2b + 3a + b$

d $ab + 3ab - 4ab$

f $2x^2 - 7 + x^2 - 3$

h $3a + b - 2a - 7b$

i $5bc - 8bc + 3$

k $-x^3 - x^2 + x^3 - 3x^2$

m $xy + y - 2xy - 3y$

j $3x^2 + x - x^2 - 2x$

l $2x - y - -x - y$

n $-2x - 4 - 3x - 6$

G

PRODUCT AND QUOTIENT SIMPLIFICATION

In this section we will simplify products like $2x \times 3x^2$ and quotients or divisions like $4x^3 \div 2x$ or $\frac{4x^3}{2x}$.

PRODUCTS

The **product** of two factors is found by multiplying them together.

Example 14

Self Tutor

Simplify:

a $2x \times 5$

b $4x \times 3x^2$

c $6x^2 \times 5x^2$

$$\begin{aligned} \mathbf{a} \quad & 2x \times 5 \\ & = 2 \times x \times 5 \\ & = 10x \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & 4x \times 3x^2 \\ & = 4 \times x \times 3 \times x \times x \\ & = 12x^3 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & 6x^2 \times 5x^2 \\ & = 6 \times x \times x \times 5 \times x \times x \\ & = 30x^4 \end{aligned}$$

With practice, you should be able to do these **mentally**. The following procedure may help you:

Consider the factors $-2x$ and $3x^2$. Their product $-2x \times 3x^2$ can be simplified by following the steps below:

Step 1: Find the product of the **signs**.

Step 2: Find the product of the **numerals** or numbers.

Step 3: Find the product of the **variables** or letters.

For $-2x$, the sign is $-$, the numeral is 2 , and the variable is x .

So, $-2x \times 3x^2 = -6x^3$

$$\begin{array}{ccccccc} - & \times & + & = & - & & \\ & & & & & \uparrow & \uparrow & \uparrow & & \\ & & & & & 2 & \times & 3 & = & 6 & & & & & x & \times & x^2 & = & x^3 \end{array}$$



EXERCISE 2G.1

1 Write the following algebraic products in simplest form:

a $x \times y$

b $x \times 3 \times y$

c $x \times y \times x$

d $a \times 2b$

e $(-x)^2$

f $x^2 \times x$

g $(-x) \times 2x$

h $(-a) \times a^2$

2 Simplify the following:

a $3a \times b$

b $3a \times b^2$

c $3ab \times 2b$

d $5ab \times 4ab$

e $(4a)^2$

f $(3b)^2 \times b^2$

g $4y \times 2y^2$

h $5b^2 \times 2b$

i $5b^2 \times b^2$

j $3b^2 \times 4b^3$

k $4x \times (-x)$

l $(-3x) \times x$

m $2x \times (-3x)$

n $(-2x) \times (-4x)$

o $(-x^2) \times 2x$

p $3x^2 \times (-6x)$

q $5 \times (-x^3)$

r $2x \times (-x)^3$

s $4d^2 \times (-d)$

t $(2x)^3$

3 **a** Find:

i $x^2 \times x^2$

ii $x^3 \times x^2$

iii $x^4 \times x^3$

iv $x^3 \times x^6$

b By considering your answers to **a**, simplify $x^m \times x^n$ where m and n are positive whole numbers.

Distributive Law

EXERCISE 4A

1 Expand and simplify:

- | | | | |
|----------------------|----------------------|----------------------|----------------------|
| a $2(x + 7)$ | b $3(x - 2)$ | c $4(a + 3)$ | d $5(a + c)$ |
| e $6(b - 3)$ | f $7(m + 4)$ | g $2(n - p)$ | h $4(p - q)$ |
| i $3(5 + x)$ | j $5(y - x)$ | k $8(t - 8)$ | l $4(7 + m)$ |
| m $6(d + e)$ | n $2(x - 11)$ | o $3(7 + k)$ | p $5(p - q)$ |
| q $4(10 - j)$ | r $7(y + n)$ | s $2(n - 12)$ | t $8(11 - d)$ |

2 Expand and simplify:

- | | | | |
|-----------------------|-----------------------|----------------------|-----------------------|
| a $9(2x + 1)$ | b $3(1 - 3x)$ | c $5(2a + 3)$ | d $11(1 - 2n)$ |
| e $6(3x + y)$ | f $5(x - 2y)$ | g $4(3b + c)$ | h $2(a - 2b)$ |
| i $7(a - 5b)$ | j $12(2 + 3d)$ | k $8(3 - 4y)$ | l $6(5b + 3a)$ |
| m $11(2x - y)$ | n $4(p + 9q)$ | o $5(a - 8b)$ | p $2(9 + 8x)$ |
| q $3(9x + y)$ | r $7(c - 9d)$ | s $6(m + 7n)$ | t $8(8a - c)$ |

Example 3

Self Tutor

Expand and simplify:

a $2y(3y + 5)$

b $2x(3 - 2x)$

$$\begin{aligned} \mathbf{a} \quad & 2y(3y + 5) \\ &= 2y \times 3y + 2y \times 5 \\ &= 6y^2 + 10y \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & 2x(3 - 2x) \\ &= 2x \times 3 + 2x \times (-2x) \\ &= 6x - 4x^2 \end{aligned}$$

3 Expand and simplify:

- | | | | |
|-------------------------|-----------------------|------------------------|-------------------------|
| a $x(x + 2)$ | b $x(5 - x)$ | c $a(2a + 4)$ | d $b(5 - 3b)$ |
| e $a(b + 2c)$ | f $a(a^2 + 1)$ | g $a^2(2 - a)$ | h $2x(3 - 4x)$ |
| i $3x(6 - x)$ | j $5x(x - 4)$ | k $4a(1 - a)$ | l $7b(b + 2)$ |
| m $(2x + 3)x$ | n $(5 - 2x)x$ | o $ab(a + b)$ | p $a^2b(3 - b)$ |
| q $mn(m - n)$ | r $ac(c - 4a)$ | s $6p(4 - 7pq)$ | t $(3k + 5l^2)k$ |
| u $(7a^2 - 5b)b$ | v $xy(x + 9y)$ | w $(7 - 4x)xy$ | x $(3t - 5s^2)t$ |

4 Expand and simplify:

a $-2(x + 2)$

b $-3(x + 4)$

c $-4(x - 2)$

d $-5(5 - x)$

e $-(a + 2)$

f $-(x - 3)$

g $-(5 - x)$

h $-(2x + 1)$

i $-3(4 - x)$

j $-4(5x - 2)$

k $-5(3 - 4c)$

l $-(x - 2)$

Example 5

Self Tutor

Expand and simplify:

a $-a(a + 7)$

b $-4b(2b - 3)$

a $-a(a + 7)$
 $= -a \times a + -a \times 7$
 $= -a^2 - 7a$

b $-4b(2b - 3)$
 $= -4b \times 2b + -4b \times (-3)$
 $= -8b^2 + 12b$

5 Expand and simplify:

a $-a(a + 1)$

b $-b(b + 4)$

c $-c(5 - c)$

d $-x(2x + 4)$

e $-2x(1 - x)$

f $-3y(y + 2)$

g $-4a(5 - a)$

h $-6b(3 - 2b)$

6 Expand and simplify:

a $3(x + 2) + 2(x + 3)$

c $4(x - 2) + 5(x - 4)$

e $5(m + 4) - 3(m - 2)$

g $3(x - 1) - 2(2x - 4)$

i $7(x + 2) + 3(2 - 4x)$

k $-5(n - 4) - 3(2n - 5)$

m $9(x + 1) + 3(2x - 3) - 15x$

b $5(x + 3) + 4(x + 3)$

d $3(2x - 7) + 2(1 - x)$

f $5(m - 3) - 2(m + 1)$

h $7(1 - x) - (3x + 2)$

j $3(4 - 3x) - 2(2x + 1)$

l $6(2y - 1) + 4(2 - y)$

n $11(2t - 1) - 3(5 - 3t) + 4$

Example 7

Self Tutor

Expand and simplify:

a $4 - 2(x + 3)$

b $8 - 3(2y - 1)$

$$\begin{aligned} \text{a} \quad & 4 - 2(x + 3) \\ & = 4 + -2(x + 3) \\ & = 4 + -2 \times x + -2 \times 3 \\ & = 4 - 2x - 6 \\ & = -2x - 2 \end{aligned}$$

$$\begin{aligned} \text{b} \quad & 8 - 3(2y - 1) \\ & = 8 + -3(2y - 1) \\ & = 8 + -3 \times 2y + -3 \times (-1) \\ & = 8 - 6y + 3 \\ & = 11 - 6y \end{aligned}$$

7 Expand and simplify:

a $3x - (2x + 1)$

d $11x - (2 - x)$

g $16 - 7(1 - 3x)$

j $7 - (1 - 2x)$

b $5 - 3(x + 2)$

e $6 - 5(1 - 2x)$

h $x + 6 - 3(4 - x)$

k $2x - (8 - 7x) + 3$

c $7 - 6(2x - 3)$

f $11 - (3 - 2x)$

i $8x + 1 - 2(3 - 2x)$

l $8 - 5(11 - 3x)$

Algebraic Substitution

Example 10

 Self Tutor

For $a = 2$, $b = -1$ and $c = 3$, evaluate:

a $3a - 2b$

b $c^2 + b$

a $3a - 2b$

$= 3 \times 2 - 2 \times (-1)$

$= 6 + 2$

$= 8$

b $c^2 + b$

$= 3^2 + (-1)$

$= 9 - 1$

$= 8$

EXERCISE 2E

1 If $a = 1$, $b = 2$ and $c = 3$, find the value of:

a $a + b$

b $2a$

c $a - c$

d $b + 2c$

e $5 - 2c$

f bc

g $ab - a$

h $2b^2$

i $2(a + b)$

j $3(c - a)$

k $(2a)^2$

l $3c^2$

2 If $k = 4$, $l = -2$, $m = 3$, and $n = -1$, evaluate:

a $2k + l$

b $2m - 5$

c $kl + n$

d $2n^2$

e $(2n)^2$

f $l^2 - m$

g k^3

h $3l - 4n$

i $k^2 - 2l$

j $l(n + 2)$

k $2(k + l)$

l $2k + 3m$

3 If $p = 3$, $q = -2$, $r = -3$ and $s = 4$, evaluate:

a p^2

b $-p^2$

c r^2s

d pqr

e pq^2

f $p^2 + q - 3$

g $pr + 2s$

h $p - q^2 + 2$

i $q^2 + r$

j $2(q + s)$

k $p(r + s)$

l $(p + q)(p - q)$

4 If $a = 2$, $b = -3$, $c = 4$, and $d = -1$, find the value of:

a $\frac{c}{a}$

b $\frac{b}{d}$

c $\frac{b+d}{c}$

d $\frac{a-d}{b}$

e $\frac{b+d}{b-d}$

f $\frac{b+2}{1-a}$

g $\frac{c}{2d}$

h $\frac{3a}{2b}$

i $\frac{a+c}{2b}$

j $\frac{-a}{b+d}$

k $\frac{a-b}{a-d}$

l $\frac{a+2c}{a-2c}$

5 For

p	q	r	s	t
-3	-2	2	4	-1

find the value of:

a $\frac{q}{r}$

b $\frac{q-r}{s}$

c $\frac{q+t}{2p}$

d $\frac{q+s}{r-t}$

e $\frac{2q}{-t}$

f $\frac{s-q}{q^2}$

g $\frac{2q}{q-s}$

h $\frac{r-t}{r-q}$

i $\frac{rs}{qt}$

j $\frac{q^2+t}{p^2}$

k $\frac{s+rt}{p}$

l $\frac{qst}{-pr}$

SOLUTIONS

EXERCISE 2F

- 1** a $x + 8$ b $x + 15$ c $p + 10$ d $2a + 12$ e $2b + 3$
f $2b$ g $3x$ h $2a + 10$ i $4x$ j x k $2x$ l $2a^2$
m $7x + 3$ n $3x^2$ o $17x - 7$ p $16x$ q $2b^2$ r $5ab$
s $3g$ t $2b - 2$
- 2** a 0 b $10n$ c $11n - 11$ d $4ab$ e $3xy$ f p^2
g $4a + 6$ h $9a$ i $3b + 7$ j $7xy$ k $5a + 2b$
l $4a^2 + 3a$ m $4x$ n 0 o $3ab + 2b^2$ p $10x - 10$
q 0 r $9x$ s $2r^2 + 2r$ t $x^2 + x + 2$ u $8y + 2$
- 3** a $11x$ b $-5x$ c $5x$ d $-11x$ e $6a$ f $4a$
g $-4a$ h $-6a$ i $m^2 + 2m$ j $-8d$ k $2d$
l $-2d$ m $3b - 3$ n $-4t$ o $-7g$ p $1 - 3m$
q $-2a + 2$ r b s $2b$ t $4b$ u $3x$
- 4** a $-a - 3$ b $4a - b$ c $-2ab - 2$ d 0
e $-x$ f $3x^2 - 10$ g $-2n - 5$ h $a - 6b$
i $3 - 3bc$ j $2x^2 - x$ k $-4x^2$ l $3x - 2y$
m $-xy - 2y$ n $-5x - 10$

EXERCISE 2G.1

- 1** a xy b $3xy$ c x^2y d $2ab$ e x^2
f x^3 g $-2x^2$ h $-a^3$
- 2** a $3ab$ b $3ab^2$ c $6ab^2$ d $20a^2b^2$ e $16a^2$
f $9b^4$ g $8y^3$ h $10b^3$ i $5b^4$ j $12b^5$
k $-4x^2$ l $-3x^2$ m $-6x^2$ n $8x^2$ o $-2x^3$
p $-18x^3$ q $-5x^3$ r $-2x^4$ s $-4d^3$ t $8x^3$
- 3** a i x^4 ii x^5 iii x^7 iv x^9 b $x^m \times x^n = x^{m+n}$

EXERCISE 2G.2

- 1** **a** x^2 **b** x^3 **c** x^3 **d** x^2 **e** $3x^2$ **f** $5x^2$
g $5x^2$ **h** $2x^3$ **i** $2x^4$ **j** $2x^3$ **k** $2x^2$ **l** $\frac{x}{2}$
m $\frac{1}{2x}$ **n** $\frac{1}{2x^2}$ **o** $\frac{1}{2x}$ **p** $\frac{1}{3x^2}$
- 2** **a** **i** x^2 **ii** x^3 **iii** x^5 **iv** x^3 **b** $\frac{x^m}{x^n} = x^{m-n}$

EXERCISE 2E

- 1** **a** 3 **b** 2 **c** -2 **d** 8 **e** -1 **f** 6
g 1 **h** 8 **i** 6 **j** 6 **k** 4 **l** 27
- 2** **a** 6 **b** 1 **c** -9 **d** 2 **e** 4 **f** 1
g 64 **h** -2 **i** 20 **j** -2 **k** 4 **l** 17
- 3** **a** 9 **b** -9 **c** 36 **d** 18 **e** 12 **f** 4
g -1 **h** 1 **i** 1 **j** 4 **k** 3 **l** 5
- 4** **a** 2 **b** 3 **c** -1 **d** -1 **e** 2 **f** 1
g -2 **h** -1 **i** -1 **j** $\frac{1}{2}$ **k** $\frac{5}{3}$ **l** $-\frac{5}{3}$
- 5** **a** -1 **b** -1 **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** -4 **f** $\frac{3}{2}$
g $\frac{2}{3}$ **h** $\frac{3}{4}$ **i** 4 **j** $\frac{1}{3}$ **k** $-\frac{2}{3}$ **l** $\frac{4}{3}$

EXERCISE 4A

- 1** **a** $2x + 14$ **b** $3x - 6$ **c** $4a + 12$ **d** $5a + 5c$
 e $6b - 18$ **f** $7m + 28$ **g** $2n - 2p$ **h** $4p - 4q$
 i $15 + 3x$ **j** $5y - 5x$ **k** $8t - 64$ **l** $28 + 4m$
m $6d + 6e$ **n** $2x - 22$ **o** $21 + 3k$ **p** $5p - 5q$
 q $40 - 4j$ **r** $7y + 7n$ **s** $2n - 24$ **t** $88 - 8d$
- 2** **a** $18x + 9$ **b** $3 - 9x$ **c** $10a + 15$ **d** $11 - 22n$
 e $18x + 6y$ **f** $5x - 10y$ **g** $12b + 4c$ **h** $2a - 4b$
 i $7a - 35b$ **j** $24 + 36d$ **k** $24 - 32y$ **l** $30b + 18a$
m $22x - 11y$ **n** $4p + 36q$ **o** $5a - 40b$ **p** $18 + 16x$
 q $27x + 3y$ **r** $7c - 63d$ **s** $6m + 42n$ **t** $64a - 8c$
- 3** **a** $x^2 + 2x$ **b** $5x - x^2$ **c** $2a^2 + 4a$ **d** $5b - 3b^2$
 e $ab + 2ac$ **f** $a^3 + a$ **g** $2a^2 - a^3$ **h** $6x - 8x^2$
 i $18x - 3x^2$ **j** $5x^2 - 20x$ **k** $4a - 4a^2$ **l** $7b^2 + 14b$
m $2x^2 + 3x$ **n** $5x - 2x^2$ **o** $a^2b + ab^2$
p $3a^2b - a^2b^2$ **q** $m^2n - mn^2$ **r** $ac^2 - 4a^2c$
 s $24p - 42p^2q$ **t** $3k^2 + 5kl^2$ **u** $7a^2b - 5b^2$
 v $x^2y + 9xy^2$ **w** $7xy - 4x^2y$ **x** $3t^2 - 5s^2t$
- 4** **a** $-2x - 4$ **b** $-3x - 12$ **c** $-4x + 8$ **d** $-25 + 5x$
 e $-a - 2$ **f** $-x + 3$ **g** $-5 + x$ **h** $-2x - 1$
 i $-12 + 3x$ **j** $-20x + 8$ **k** $-15 + 20c$ **l** $-x + 2$
- 5** **a** $-a^2 - a$ **b** $-b^2 - 4b$ **c** $-5c + c^2$
 d $-2x^2 - 4x$ **e** $-2x + 2x^2$ **f** $-3y^2 - 6y$
 g $-20a + 4a^2$ **h** $-18b + 12b^2$
- 6** **a** $5x + 12$ **b** $9x + 27$ **c** $9x - 28$ **d** $4x - 19$
 e $2m + 26$ **f** $3m - 17$ **g** $-x + 5$ **h** $-10x + 5$
 i $-5x + 20$ **j** $-13x + 10$ **k** $-11n + 35$ **l** $8y + 2$
m 0 **n** $31t - 22$
- 7** **a** $x - 1$ **b** $-1 - 3x$ **c** $25 - 12x$ **d** $12x - 2$
 e $1 + 10x$ **f** $8 + 2x$ **g** $9 + 21x$ **h** $4x - 6$
 i $12x - 5$ **j** $6 + 2x$ **k** $9x - 5$ **l** $-47 + 15x$